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Cosmology based on $f(R)$ gravity with $\mathcal{O}(1)$ eV sterile neutrino

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Abstract. We address the cosmological role of an additional $\mathcal{O}(1)$ eV sterile neutrino in modified gravity models. We confront the present cosmological data with predictions of the FLRW cosmological model based on a variant of $f(R)$ modified gravity proposed by one of the authors previously. This viable cosmological model which deviation from general relativity with a cosmological constant Λ decreases as R^{-2n} for large, but not too large values of the Ricci scalar R (while no Λ is introduced by hand at small R) provides an alternative explanation of present dark energy and the accelerated expansion of the Universe (the case $n = 2$ is considered in the paper). Various up-to-date cosmological data sets exploited include measurements of the cosmic microwave background (CMB) anisotropy, the CMB lensing potential, the baryon acoustic oscillations (BAO), the cluster mass function and the Hubble constant. We find that the CMB+BAO constraints strongly restrict the sum of neutrino masses from above. This excludes values of the model parameter $\lambda \sim 1$ for which distinctive cosmological features of the model are mostly pronounced as compared to the Λ CDM model, since then free streaming damping of perturbations due to neutrino rest masses is not sufficient to compensate their extra growth occurring in $f(R)$ modified gravity. Thus, in the gravity sector we obtain $\lambda > 8.2$ (2σ) with the account of systematic uncertainties in galaxy cluster mass function measurements and $\lambda > 9.4$ (2σ) without them. At the same time in the latter case we find for the sterile neutrino mass $0.47 \text{ eV} < m_{\nu, \text{sterile}} < 1 \text{ eV}$ (2σ) assuming that the sterile neutrinos are thermalized and the active neutrinos are massless, not significantly larger than in the standard Λ CDM with the same data set: $0.45 \text{ eV} < m_{\nu, \text{sterile}} < 0.92 \text{ eV}$ (2σ). However, a possible discovery of a sterile neutrino with the mass $m_{\nu, \text{sterile}} \approx 1.5 \text{ eV}$ motivated by various anomalies in neutrino oscillation experiments would favor cosmology based on $f(R)$ gravity rather than the Λ CDM model.

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1 Introduction

The fact that the present Universe is undergoing an accelerated expansion is firmly established by numerous observational data. The standard Λ -Cold-Dark-Matter (Λ CDM) cosmological model can explain the cosmic acceleration at expense of introducing a new fundamental physical parameter, the cosmological constant Λ . Its observed value is much smaller than any other energy scale of the fundamental physical interactions, that presents a great challenge for the theoretical elementary particle physics.

A stage similar to the present accelerated expansion one, dubbed inflation, is believed to happen in the very early Universe. We know that the source of inflation may not be identical to the cosmological constant since the inflaton field – primordial Dark Energy (DE) – was evolving and unstable. This qualitative analogy provides with an additional argument in favor of non-stationary models of the present DE alternative to Λ .

In this paper, we consider so-called $f(R)$ gravity (see e.g. [1–4] for reviews, and [5–7] for the first viable cosmological models relevant for the present Universe), which modifies General Relativity (GR) by replacing the scalar curvature (the Ricci scalar) R with a new phenomenological function $f(R)$ in the Einstein-Hilbert action. It represents a special case of more general scalar-tensor Brans-Dicke theory [8] with the Brans-Dicke parameter $\omega_{BD} = 0$ [9]. Cosmological models based of this modified gravity can explain the present cosmic acceleration without introducing Λ , so we can put $f(0) = 0$. There is a new scalar degree of freedom in the gravity sector dubbed *scalaron* [10] responsible for extra growth of matter density perturbations in $f(R)$ models. That is the most dramatic difference from the Λ CDM model which we have to cope with.

For an $f(R)$ model to be phenomenologically viable and theoretically consistent and to solve the above difficulties, it should satisfy a list of viability conditions. First, an $f(R)$ should satisfy the necessary conditions in the region of relevant values of R :

$$f'(R) > 0, \quad f''(R) > 0, \quad (1.1)$$

hereinafter prime denotes a derivative with respect to argument R . The first condition in (1.1) means that the gravity is attractive and graviton is not a ghost. The second condition guarantees that scalaron is not a tachyon both in the Minkowski space-time and in the regime of small deviations from GR. Note that it is necessary to keep conditions (1.1) for all values of R during the matter- and radiation-dominated stages in order to avoid the Dolgov–Kawasaki instability [11]. If one wants to incorporate the early-time inflation, the range of R has to be extended accordingly, see discussion in Sec. 2.

Second, the existence of the new additional degree of freedom imposes a number of special conditions on the functional form of $f(R)$ for $R \gg R_0$ [7]:

$$|f(R) - R| \ll R, \quad |f'(R) - 1| \ll 1, \quad f''(R)R \ll 1, \quad (1.2)$$

where R_0 is the present Ricci curvature. These conditions guarantee the correct Newtonian limit for the matter-dominated stage in the past and smallness of non-GR corrections to a space-time background metric for a more general background of compact astrophysical objects in the present Universe. The third condition in (1.2) implies that the Compton wavelength of the scalaron field is much less than the curvature radius of the background metric. It ensures the absence of extra growth of the matter perturbations in high-density regions that is necessary to satisfy local gravity constraints (LGC). On the other hand, in principle, this condition may be violated at $R \sim R_0$ due to dependence of the effective scalaron mass on R which, in turn, is determined by the matter density in the regime of small deviations from GR. In cosmology, such effect is often called the chameleon mechanism [5], though it occurs in many other areas of physics, too, c.f. the well-known dependence of the plasmon mass on density in plasma physics. It is important for understanding of behavior of the cosmological perturbations in $f(R)$ gravity (see discussion in Sec. 3).

If the above constraints are satisfied, cosmological models based on $f(R)$ gravity can describe FLRW background expansion history similarly to that of the Λ CDM model. However, inhomogeneous metric fluctuations evolve differently. In particular, matter density perturbations grow faster on scales smaller than the Compton wavelength of the scalaron field that occurs at recent redshifts. One needs something to compensate for this extra growth. For instance, neutrino rest masses can do this job. The free streaming of the massive neutrinos suppresses the structure formation on small scales. Hence, if one adjusts the neutrino masses in the $f(R)$ gravity, the net result can be zero, because the $f(R)$ modification and neutrino masses play opposite roles in the evolution of matter density perturbations on small scales [12].

The same mechanism works for a $O(1)$ eV sterile neutrino added to the Standard Model of elementary particles. If mixing with the active neutrino is not extremely small, the sterile neutrinos are produced in the primordial plasma and get thermalized in the early Universe before the active neutrino decoupling. While relativistic they contribute to the radiation component as one additional neutrino species. In particular, this component increases the Universe expansion rate at the Big Bang Nucleosynthesis (BBN) epoch. The recent reanalysis of the primordial helium abundance permits the existence of one extra neutrino species [13]: an effective number of neutrinos is $N_{eff} = 3.58 \pm 0.40(2\sigma)$ for the neutron lifetime $\tau_n = 880.1 \pm 1.1$ s, while the standard three active neutrinos give $N_{eff} = 3.046$ [14].

The light sterile neutrinos are interesting because of anomalous results obtained by several neutrino oscillation experiments which do not fit to the three neutrinos oscillation pattern and ask for one (or two) more light neutrinos [15]. In particular, the so-called gallium anomaly observed by GALLEX [16, 17] and SAGE [18, 19] experiments is nicely explained

as the electron neutrino oscillations into sterile neutrino of 1.5 eV mass [20]. The reactor antineutrino anomaly (disappearance of electron antineutrinos from nuclear reactors) [21, 22] is consistent with sterile neutrino of the same mass, while account for other anomalies from accelerator experiments shifts the mass in the combined fit to 1 – 1.3 eV, see [15] for details.

In this paper we confront the most recent observational data with predictions of cosmology based on $f(R)$ gravity and supplemented with light sterile neutrinos. In Sec. 2 we present the $f(R)$ cosmological model and describe the cosmological background (homogeneous Universe) evolution. In Sec. 3 we consider matter density perturbations. We fit the model predictions to the observational data in Sec. 4. For the $f(R)$ model we outline the allowed region in the model parameter space using modified MGCAMB and CosmoMC. We find that 1.5 eV sterile neutrino is better consistent with DE models based on the $f(R)$ gravity rather than with the standard Λ CDM. Among all the cosmological data used, the most important appear to be those from observations of galaxy clusters which trace the evolution of density perturbations. In contrast to the paper [23] where the same problem was studied, we do not use the power spectrum of matter density perturbations obtained from galaxy clustering data to avoid problems with the bias parameter. Instead, we employ more recent and accurate results on the abundance of galaxy clusters.

2 Background Universe

We define $f(R)$ gravity by the following action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m, \quad (2.1)$$

where $\kappa^2/(8\pi) \equiv G$ is the Newton gravitational constant and S_m is the action of matter fields all minimally coupled to gravity.

We take the $f(R)$ model [7]

$$f(R) = R + \lambda R_s \left[\left(1 + \frac{R^2}{R_s^2} \right)^{-n} - 1 \right], \quad (2.2)$$

where n, λ, R_s are model parameters. Strictly speaking, the model (2.2) has to be modified at very large values of R , e.g. supplemented with the term $R^2/6M^2$ borrowed from the inflationary model [10] where M is the inflaton mass. This term solves problems discussed in Ref. [24]: the scalaron mass exceeding the Planck mass and a weak curvature singularity at some finite time in the past. The value of M should be sufficiently large in order to pass laboratory and Solar system tests of gravity, namely one has $M > 10^{-2.5}$ eV according to Cavendish-type experiment [25]. However, any type of inflation in the early Universe (driven by either scalaron or another field) imposes a much higher upper limit on M , only several orders of magnitude less than the Planck mass. In particular, in the latter case, one has $M \gg H_{inf}$ where H_{inf} is the Hubble parameter at the end of inflation. As a result, this high- R correction to $f(R)$ becomes negligible for the low- R cosmology we are interested in. Also, the function (2.2) should be modified and conditions (1.1) should be revised for $R < R_0$ including the region $R < 0$ (R becomes negative during post-inflationary evolution, see [24] for detailed study of this issue). Once more, this change does not affect our case where $R \geq R_0$.

2.1 Field equations

We derive field equations by varying the action (2.1) with respect to space-time metric¹ $g_{\mu\nu}$

$$f' R_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) f' = \kappa^2 T_{\mu\nu}^{(M)}, \quad (2.3)$$

where $R_{\mu\nu}$ is the Ricci tensor, ∇_μ is the covariant derivative associated with the metric $g_{\mu\nu}$, $\square \phi \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu \phi$ and $T_{\mu\nu}^{(M)}$ is the matter stress-energy tensor.

We obtain two gravitational field equations from diagonal elements of (2.3):

$$3H^2 f' - \frac{1}{2} (R f' - f) + 3H \dot{f}' = \kappa^2 \rho_m, \quad (2.4)$$

$$(\dot{H} + 3H^2) f' - \frac{1}{2} f - \ddot{f}' - 2H \dot{f}' = \kappa^2 P_m, \quad (2.5)$$

where $H = \dot{a}/a$ is the Hubble parameter, ρ_m and P_m are the energy density and pressure of non-relativistic matter ($P_m = 0$), respectively, and hereafter dot denotes derivative with respect to the cosmic time t .

We can rewrite (2.3) in the following Einsteinian form,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 \left(T_{\mu\nu}^{(M)} + T_{\mu\nu}^{(DE)} \right), \quad (2.6)$$

where

$$\kappa^2 T_{\mu\nu}^{(DE)} \equiv \frac{1}{2} g_{\mu\nu} (f - R) - (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) f' + R_{\mu\nu} (1 - f'). \quad (2.7)$$

Then from (2.4), (2.5) and (2.7) we obtain the effective DE density ρ_{DE} and pressure P_{DE} :

$$\kappa^2 \rho_{DE} = -3H \dot{f}' + 3(H^2 + \dot{H})(f' - 1) - \frac{1}{2} (f - R), \quad (2.8)$$

$$\kappa^2 P_{DE} = \ddot{f}'(R) + 2H \dot{f}' - (3H^2 + \dot{H})(f' - 1) + \frac{1}{2} (f - R). \quad (2.9)$$

Then we define the equation-of-state parameter ω_{DE} for the DE component by

$$\omega_{DE} \equiv \frac{P_{DE}}{\rho_{DE}} = -1 + \frac{2\dot{H}(f' - 1) - H \dot{f}' + \ddot{f}'}{-3H \dot{f}' + 3(H^2 + \dot{H})(f' - 1) - (f - R)/2}. \quad (2.10)$$

2.2 Numerical calculation

We solve equation (2.4) numerically in the $f(R)$ model (2.2). To find the exact value of R_s , we require that the density parameter for non-relativistic matter equals $\Omega_m = 0.3$ and the Hubble parameter is $H_0 = 72$ km/s/Mpc in the present Universe.

Figure 1 depicts the evolution of ω_{DE} as a function of redshift z for reference values of n and λ . Remarkably, the condition of stability of the future de Sitter asymptotic solution [26] imposes constraints on the free parameter λ . For instance, if $n = 2$ then $\lambda > 0.94$, if $n = 3$ then $\lambda > 0.73$, and if $n = 4$ then $\lambda > 0.61$ according to [27]. The parameter ω_{DE} approaches the constant value $\omega_{DE} = -1$ as we increase λ for fixed n , that is in the Λ CDM-like limit. For minimal allowed values of λ , deviation of the DE equation-of-state parameter from the value $\omega_{DE} = -1$ for redshifts $z \lesssim 2$ is consistent with recent observational data [28].

For the background metric, we find that the phantom boundary crossing ($\omega_{DE} = -1$) occurs at small redshifts $z \lesssim 1$. This phantom crossing is not peculiar to the specific choice of the function (2.2). It is a generic feature of all models which obey $f''(R) > 0$.

¹We use the sign conventions in which the metric given by $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$, where $a(t)$ is the scale factor.

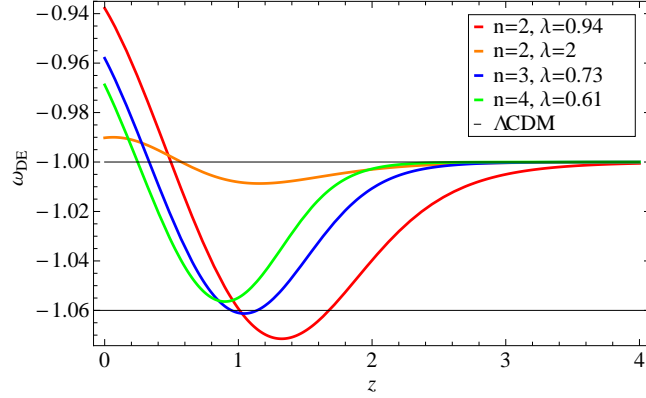


Figure 1. Evolution of the equation-of-state parameter ω_{DE} for DE in the $f(R)$ model (2.2).

2.3 The First Iteration Approach

Numerical solution of eq. (2.4) is not convenient for implementation in the computer simulation programme MGCAMB we would like to use. An alternative way is to use for the MGCAMB variables the expressions derived within the $f(R)$ gravity (i.e. eq. (2.10) for ω_{DE} instead of $\omega_{DE} = -1$, etc.) with the scale factor $a(t)$ solving the Λ CDM equations. It is the so-called iteration method [7]. We consider the first iteration only and call it the First Iteration Approach (FIA). It allows us to catch the leading deviation of background evolution in the $f(R)$ model from that in the Λ CDM model (that it is also necessary to determine the change in the Integrated Sachs–Wolfe effect [29]) and compute matter density perturbations in Sec. 4 more precisely.

Curve (a) in Fig. 2 represents the modulus of deviation of the equation-of-state parameter

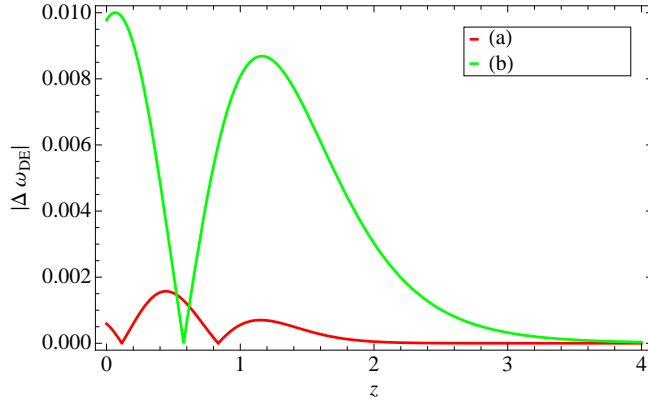


Figure 2. Modulus of deviation of the equation-of-state parameter ω_{DE} derived by the FIA from that derived by numerical calculation (a), modulus of deviation of the constant value $\omega_{DE} = -1$ inherited by the Λ CDM model from the precise calculation of the equation-of-state parameter ω_{DE} (b).

ω_{DE} for DE derived by FIA from that derived by the straightforward numerical calculation described above for $n = 2$, $\lambda = 2$ and the same values of Ω_m and H_0 (the same present epoch) adopted in Fig. 1. The deviation do not exceed 0.2 %. Obviously, it is even smaller for

larger values of n or λ . Curve (b) in Fig. 2 depicts the modulus of deviation of the constant value $\omega_{DE} = -1$ in the Λ CDM model from the numerical calculation of the equation-of-state parameter ω_{DE} for the same values of parameters and the same present epoch. Clearly, at $\lambda = 2$ the FIA yields a more accurate estimate of ω_{DE} than one gets adopting the background behaviour of the Λ CDM model. Moreover, for values $\lambda > 1.1$ such approach works better than the approximation within Λ CDM. Actually, for values $\lambda > 1.5$ the deviation of the FIA results from the numerical solution is less than 1 %. This precision is enough to extract values of cosmological parameters with the percent accuracy, so we adopt it in what follows.

3 Matter perturbations

3.1 Linear evolution of the matter perturbations

We turn to the linear evolution of matter density fluctuations in cosmological models based on $f(R)$ gravity. We define metric perturbations by

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)\delta_{ij}dx^i dx^j. \quad (3.1)$$

For sub-Hubble modes in the quasi-static approximation, the equations for the matter density contrast $\delta \equiv \delta\rho/\rho$ and the gravitational slip in $f(R)$ gravity read [3]

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}(t, k)\rho\delta = 0, \quad (3.2)$$

$$\frac{\Psi}{\Phi} = \eta(t, k), \quad (3.3)$$

where

$$G_{eff}(t, k) \equiv \frac{G}{f'} \frac{1 + 4\frac{k^2}{a^2} \frac{f''}{f'}}{1 + 3\frac{k^2}{a^2} \frac{f''}{f'}}, \quad (3.4)$$

$$\eta(t, k) \equiv \frac{1 + 2\frac{k^2}{a^2} \frac{f''}{f'}}{1 + 4\frac{k^2}{a^2} \frac{f''}{f'}}. \quad (3.5)$$

In the quasi-GR regime ($f' \approx 1$), the effective scalaron mass is given by [3]

$$M_s^2 \approx \frac{1}{3f''(R)}. \quad (3.6)$$

There are two different extreme regimes of evolution of the density fluctuations: $M_s \gg k/a$ and $M_s \ll k/a$. The former regime corresponds to $G_{eff} \approx G$ (thus, the evolution of δ mimics that in GR), whereas the latter corresponds to $G_{eff} \approx 4G/3$ (this is the case of amplification of matter density perturbations mentioned in Introduction). Consequently, the effective gravitational "constant" increases up to 33 %, independently of the functional form of $f(R)$.

3.2 Beyond the linear regime and LGC

Matter perturbations of characteristic scale ℓ exhibit an extra growth [5] for small $\ell < \lambda_c$ ($R = \kappa^2 \rho_\ell$), where $\lambda_c = M_s^{-1}$ is the Compton wavelength of the scalaron field, ρ_ℓ is the local energy density of a particular structure. At smaller scales referring to given astrophysical objects, the local tests of $f(R)$ gravity become important.

Scalaron has to be heavy enough and unobservable to pass both the cosmological and the Solar system tests. The chameleon mechanism takes care of LGC for compact objects in the present Universe. Scalaron mass M_s depends on a local value of the Ricci scalar R arising in a local structure of the characteristic size ℓ . Cavendish-type experiments with the values $\rho_\ell \sim 10^{-12}$ g/cm³ and $\ell \sim 10^{-2}$ cm [30] give a fairly weak constraint on model parameters n, λ . In the case of Solar system tests, for minimal allowed value $\rho_\ell \sim 10^{-24}$ g/cm³ and length-scale $\ell = 1$ Au = 1.5×10^{13} cm we obtain the stronger constraint $n \geq 2$. It seems that investigating larger objects – galaxies and galaxy clusters – one can impose much stronger constraints because the energy density on the outskirts of halos can be estimated by the present cosmic background matter density $\rho_\ell \sim 10^{-29}$ g/cm³. Really, the involved linear effects begin to dominate on scales with such matter density. Therefore we are interested in a violation of the Compton condition $\ell < \lambda_c$ for linear perturbations, since galaxy tests do not lead to tighter constraints. Moreover, as it was shown in [5], the thin shell bound for a galaxy is overly restrictive. To summarize different LGC, the range $n \geq 2$ is already sufficiently large to pass the Solar system and other tests.

On the other hand, if we want to have an additional growth of linear perturbations in the present Universe, we should put $M_s \gtrsim H_0$ which corresponds to the Compton wavelength less but not much less than the size of a visible part of the Universe. This condition together with the aforementioned LGC give finally the constraint $n \gtrsim 2$. Indeed, in the case $n = 2$ for the minimal available value $\lambda = 0.94$, we obtain $M_s/H_0 = 3.6$ at the present epoch described by $\Omega_m = 0.3$, $H_0 = 72$ km/s/Mpc.

It is convenient to use the dimensionless Compton wavelength squared in Hubble units today $B_0 = (f''/f')(dR/d \ln H)|_{t=t_0}$ [31], the so-called deviation index, which plays an important role in the both cosmological and local tests of $f(R)$ models. We calculate this index for different values of λ at fixed $n = 2$ for the same present epoch described in the previous paragraph. We find $B_0 = 1.92 \times 10^{-1}$, 5.8×10^{-5} , 2.4×10^{-5} and 1.5×10^{-6} for $\lambda = 0.94, 8, 10$ and 20 , respectively. Noteworthy, we calculate B_0 within the gravity law (2.2) which corresponds to the scalaron mass squared behavior $M_s^2 \propto R^6$, whereas for the often used Bertschinger–Zukin (BZ) parametrization one has $M_s^2 \propto R^2$ [32] for large values of the scalar curvature R , so that the effective cosmological constant grows logarithmically with R .

4 Parameter constraints

We carry out the Markov Chain Monte Carlo (MCMC) analysis for the Λ CDM model and the $f(R)$ gravity described by (2.2) with and without one massive sterile neutrino which is taken to be thermalized and shares the same temperature as the active neutrinos. We neglect masses of the three standard neutrinos compared to that of one sterile neutrino. We have modified the MGCAMB [33, 34] that allows to implement $f(R)$ gravity by adopting (3.4) and (3.5). We change the background evolution equations as provided by the FIA (see subsection 2.3) catching the first nonvanishing correction to the background evolution in the Λ CDM model. We plugged the above modified MGCAMB code into CosmoMC [35, 36] to constrain the model parameters. We use six standard free fitting parameters: the density parameters for baryon matter $\Omega_b h^2$ and for cold dark matter without neutrino $\Omega_c h^2$, the sound horizon angle $\theta_* \equiv 100 r_s / D_A(z_*)$, the optical depth τ , the scalar spectral index n_s and the amplitude of the primordial power spectrum $\Delta_{\mathcal{R}}^2$. Occasionally, we add extra fitting parameters such as the total mass of three active neutrinos $\sum m_\nu$ or the mass of one sterile neutrino $m_{\nu, \text{sterile}}$, so that the relative contribution of the dark matter to the present energy density is $\Omega_{DM} = \Omega_c + \Omega_\nu$.

When we work with modified gravity, we consider λ as a fitting parameter. We fix another parameter of $f(R)$ gravity as $n = 2$ because it is the minimal integer value for which the effect of the density perturbation enhancement in the linear regime is the most pronounced (see subsection 3.2).

Performing numerical calculations, we find the regions of parameter space consistent with cosmological data at the 65 % and 95 % confidence levels and outline them on plots presented below.

4.1 Cosmological Data

In our analysis we use different data sets. The first set is the measurement of the CMB temperature power spectrum from the one-year data release of the Planck satellite [37] supplemented with the low- ℓ polarization measurements from the nine-years observations of the WMAP satellite [38, 39]. This data set is designated below as 'Planck'. We extend this data set with CMB measurements at high- ℓ by the Atacama Cosmology Telescope (ACT) [40] and the South Pole Telescope (SPT) [41–43]. We refer to these measurements below as *ePlanck*.

We also include different measurements of baryon acoustic oscillation (BAO) including the LOWZ [44] and CMASS [45] samples of BOSS corresponding to SDSS DR11 in the redshift range $0.15 < z < 0.43$ and $0.43 < z < 0.7$, respectively, and also the 6dF Galaxy Survey [46] corresponding to $z = 0.106$. These BAO data sets do not overlap, and therefore we can use them together. We refer to this set combination as *BAO*.

In addition, we use the Hubble constant measurement [47] mentioned below as H_0 and the full-sky lensing potential map [48] called *LENS*.

Finally, we use observations of galaxy clusters. Data on cluster mass function measurements are taken from [49, 50] using the likelihood data described in [51]. In this study, a sample of 86 massive galaxy clusters in the ranges $z < 0.2$ and $z \approx 0.4 - 0.9$ with masses measured with about 10 % accuracy by the *Chandra* X-ray telescope was used to determine the cluster mass function (the subsample of distant massive clusters was taken from the *400d* X-ray galaxy cluster survey [52]). Likelihoods were obtained for the DE model with a constant in time parameter ω_{DE} . We take the results for $\omega_{DE} = -1$. Deviation of an effective time-dependent ω_{DE} from the value $\omega_{DE} = -1$ is below 0.1 % for $\lambda > 3.6$. As is shown below, this choice of λ is justified. We refer to this data as *CL*.

4.2 Results and discussion

At first, we compare the Λ CDM and $f(R)$ models with three active neutrinos (among them only one is taken massive) using the *ePlanck+BAO+LENS+H₀* data set. From Fig. 3 we see that $f(R)$ gravity does not relax the upper limit on neutrino mass as compared to the Λ CDM model. On the rightmost panel in Fig. 3 we see the effect of $f(R)$ gravity on the growth of matter density perturbations. Clearly, the enhancement of the σ_8 value is not restricted because the data set that constrains the structure formation is not included yet. A notable feature of modified gravity consists in an apparent peak of the posterior probability distribution at low values of λ which corresponds to the most probable growth rate of the structure formation. The reason is that the sensitivity of the CMB multipole spectrum to $f(R)$ models is mainly due to the late ISW effect changing low multipoles according to [53, 54]. Then the strong modification of the density perturbation evolution on linear scales provides better parameter convergence. This tendency is in full compliance with the recent results of [55]. Moreover, we reproduce the neutrino mass constraint from that article in the limit of negligible deviation from GR (high values of B_0) according to the first and the third panels

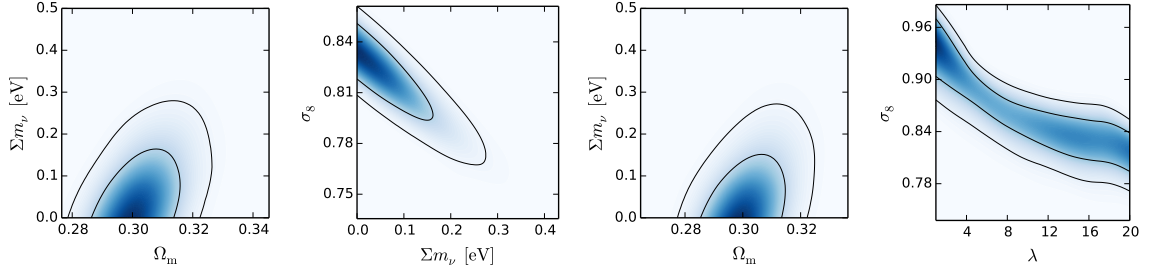


Figure 3. Constraints for the Λ CDM model in the $\sum m_\nu$ - Ω_m , σ_8 - $\sum m_\nu$ planes (two left panels) and for $f(R)$ gravity in the $\sum m_\nu$ - Ω_m , σ_8 - λ planes (two right panels) assuming one massive and two massless active neutrinos within the $ePlanck+BAO+LENS+H_0$ data set.

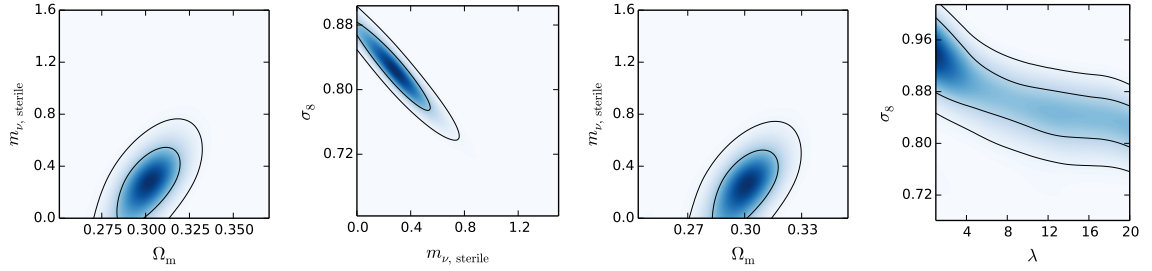


Figure 4. Constraints for the Λ CDM model in the $m_{\nu, \text{sterile}}$ - Ω_m , σ_8 - $m_{\nu, \text{sterile}}$ planes (two left panels) and for $f(R)$ gravity in the $m_{\nu, \text{sterile}}$ - Ω_m , σ_8 - λ planes (two right panels) assuming one massive sterile and three massless active neutrinos within the $ePlanck+BAO+LENS+H_0$ data set.

in Fig. 3. Remarkably, here the strong constraint on neutrino mass is obtained irrespective of the evolution history of linear perturbations.

In principle, the introduction of massive sterile species can improve the situation and leads to a weaker mass constraint. The results are shown in Fig. 4 for different models: Λ CDM and $f(R)$ gravity. Indeed, the value of (sterile) neutrino mass increases as compared to the case with only three active neutrinos. This can be understood as a necessity to keep a constant redshift of the matter-radiation equality, z_{eq} which is determined from CMB observations as explained in [56]. Nevertheless, the extra growth of linear perturbations in modified gravity at low values of R cannot be canceled by massive neutrinos, contrary to what was suggested in [23]. The reason is that the sterile neutrino mass is constrained quite strongly for any evolution history of the linear perturbations by the CMB+BAO data only.

In order to check the growth of σ_8 , we use the galaxy cluster data set. From Fig. 5 we see that the galaxy cluster data constrain σ_8 at lower values as compared to the Planck CMB data [57]. This tension can be resolved with massive neutrinos introduced into the cosmological model, which suppress the matter density fluctuations growth through the free-streaming effect discussed above. The tension was first observed with pre-Planck data [58, 59], and it is even more prominent when the Planck CMB and SZ clusters data are used, see, e.g., [57, 60–62]. This tendency is also reflected on the plot of the second panel in Fig. 5.

According to the plot on the third panel in Fig. 5 $f(R)$ gravity leads to slight degeneracy between sterile neutrino mass and the single free parameter of the modified gravity model λ . While structures in the Universe grow faster for smaller λ , the value of sterile neutrino mass has to be increased to compensate for the extra growth of perturbations at small scales in

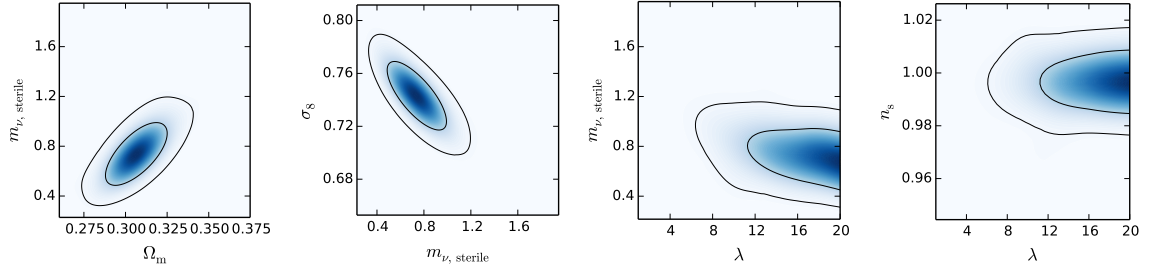


Figure 5. Viable regions in the parameter space for $f(R)$ gravity (with background evolution according to the FIA in the $m_{\nu, \text{sterile}}-\Omega_m$, $\sigma_8-m_{\nu, \text{sterile}}$, $m_{\nu, \text{sterile}}-\lambda$, $n_s-\lambda$ planes (panels from left to right) assuming one massive sterile and three massless active neutrinos within the $ePlanck+BAO+LENS+H_0+CL$ data set.

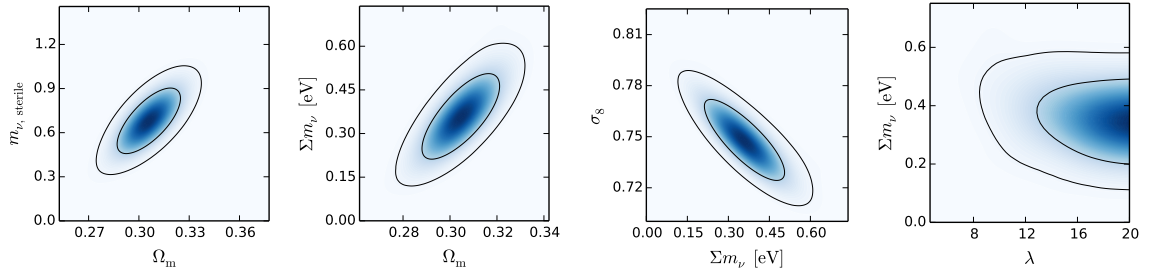


Figure 6. Posterior distributions for $m_{\nu, \text{sterile}}$ and Ω_m (the first panel) in the Λ CDM model with one massive sterile neutrino (assuming the active neutrinos are massless) and constraints on pairs $\Sigma m_{\nu}-\Omega_m$, $\sigma_8-\Sigma m_{\nu}$, $\Sigma m_{\nu}-\lambda$ (the other panels) in $f(R)$ gravity with one massive and two massless active neutrinos within the $ePlanck+BAO+LENS+H_0+CL$ data set.

modified gravity. But the sterile neutrino mass is tightly constrained from the $ePlanck+BAO$ data set. As a result, not much place for the extra growth remains after the implementation of $f(R)$. Indeed, the marginalized constraint on the sterile neutrino mass within $f(R)$ gravity is $0.47 \text{ eV} < m_{\nu, \text{sterile}} < 1 \text{ eV}$ (2σ) in contrast to $0.45 \text{ eV} < m_{\nu, \text{sterile}} < 0.92 \text{ eV}$ (2σ) in Λ CDM with three active neutrinos taken massless, according to the leftmost panels in Fig. 5 and Fig. 6. Our constraint on the sterile neutrino mass is conservative because in reality the minimal sum of active neutrino masses is nonzero – it is either 0.05 eV or 0.1 eV corresponding to the normal and inverted hierarchies.

In addition, the rightmost panel in Fig. 5 shows that after introducing one sterile neutrino $f(R)$ gravity does not change the value of the scalar spectral index n_s as compared to Λ CDM [57]. We note that the region of large values of λ corresponds approximately to the same structure formation as in Λ CDM: the results presented in Fig. 5 match in this limit similar results in Λ CDM.

Thanks to strong restrictions on σ_8 from galaxy cluster observations, we can explore the function $f(R)$ by getting a constraint on the model fitting parameter λ . Namely, from Fig. 5 we find $\lambda > 9.4$ (2σ) in the case of the fourth massive sterile neutrino and others taken massless. It implies a restriction on the deviation index at present, $B_0 < 3.1 \times 10^{-5}$ (2σ) for $\Omega_m = 0.3$ and $H_0 = 72 \text{ km/s/Mpc}$. When the systematic uncertainty of the cluster mass function $\delta M/M \approx 0.09$ [50] is included in the likelihood functions, the constraints are relaxed: $\lambda > 8.2$ (2σ) and $B_0 < 5.3 \times 10^{-5}$ (2σ). We can get restrictions on λ and B_0 in the Universe

with the three active neutrinos, too, assuming one is massive: $\lambda > 10.8$ (2σ) from Fig. 6 which corresponds to $B_0 < 1.8 \times 10^{-5}$ (2σ) without taking the systematic uncertainty of the galaxy cluster measurements into account and $\lambda > 9.6$ (2σ), $B_0 < 2.8 \times 10^{-5}$ (2σ) with that. Using these constraints on the parameter λ , we check that the use of FIA (see subsection 2.3) and galaxy clusters data set (see subsection 4.1) are justified. The obtained above bounds can be easily transformed to constraints on the present scalaron mass which determines the strength of $f(R)$ gravity. For one massive sterile neutrino we obtain $M_s/H_0 > 194$ (2σ) with the cluster systematics and $M_s/H_0 > 255$ (2σ) without it; in the case of three active neutrinos we get $M_s/H_0 > 266$ (2σ) and $M_s/H_0 > 336$ (2σ), respectively.

We use the galaxy cluster data to get rid of degeneracy between the massive sterile neutrino and modified gravity. The cluster mass function has been extracted from the cluster data assuming the Λ CDM-like setup for structure formation [63], so that the change in gravity strength inherent in $f(R)$ model (3.4) remains unaccounted. In particular, the modified gravity impact on the dynamics of matter streaming from infall region to the cluster halo in recent epoch is neglected. The median mass at all redshifts in the galaxy clusters data set is near $M_{500} = 2.5 \times 10^{14} h^{-1} M_\odot$ that corresponds to structures which were generated from the comoving critical scale [49] $8 h^{-1}$ Mpc. Such scale is rather big and $f(R)$ linear modification of density growth is quite important on these scales. For example, cosmological macrostructures, such as filaments which feed clusters with extra matter at $z \lesssim 1$, have moderate density contrast $\delta\rho/\rho \approx 2 - 3$ and the effect of modified gravity on this structure formation is quite noticeable (the present size of cosmological filaments can be estimated as 10 Mpc). Therefore we have to find the range of λ where the approach used in [63] is still valid.

According to $f(R)$ gravity, the growth of linear density perturbations is extensive on scales below the Compton wavelength of the scalaron field with the mass given by (3.6). On the contrary, light neutrinos play restrictive role to $f(R)$ gravity by damping the structure formation most efficiently on scales below the free-streaming (Jeans) length [64]

$$\lambda_{FS} = 7.7 \frac{1+z}{\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}} \left(\frac{1 \text{ eV}}{m_\nu} \right) h^{-1} \text{ Mpc}. \quad (4.1)$$

In Fig. 7 we compare the Compton wavelength of scalaron for the rather high value of $\lambda = 8$, the free-streaming wavelength of neutrino for various neutrino masses and the median critical size of cluster which is used in the galaxy cluster measurements at low redshifts $z < 1$ assuming $\Omega_m = 0.3$ and $h = 0.72$. As clearly seen from Fig. 7, for the galaxy cluster formation the free-streaming effect is maximal during all evolution for all interesting values of neutrino masses. The situation with the $f(R)$ critical scale is more complicated. We see that in the epoch $z \approx 1$ when active formation of the largest cosmological structures (galaxy clusters and filaments) begins, the scalaron Compton length exceeds the critical cluster size. Approximately from this time the Λ CDM approximation of galaxy cluster formation adopted for galaxy cluster data in our case must be corrected for the increase of gravity strength (3.4). This picture corresponds to $\lambda = 8$. If we take modified gravity with more pronounced effects, i.e. decrease λ , the Compton wavelength increases too and the impact of gravity modification on the structure formation starts earlier and extends to smaller structures. If we describe the formation of galaxy clusters with computer simulation properly accounting for the $f(R)$ gravity impact, the value of σ_8 has to be reduced because the number of galaxy clusters in the Universe is fixed by the galaxy cluster data set. For this reason, the constraint on parameter

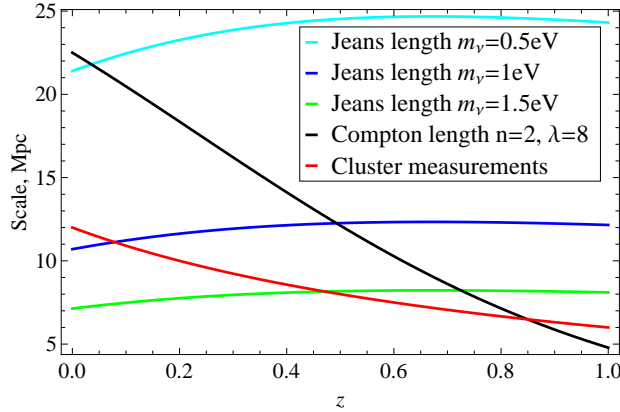


Figure 7. The Jeans scale of neutrino free-streaming and the Compton length of scalaron ($n = 2$, $\lambda = 8$) as functions of redshift z for various values of neutrino mass. Below the lines called *Jeans length*, perturbations are suppressed by the neutrino free streaming. Below the line named *Compton length*, the growth of perturbations is enhanced due to modified gravity. Clusters collect matter from regions of the size below the line labeled *Cluster measurements*.

λ obtained above from the galaxy cluster mass function derived in the Λ CDM-like analysis is conservative.

Recently a number of papers have been issued where the massive neutrinos are exploited to suppress extra growth of matter density perturbations at small scales occurring in various $f(R)$ models. For example, while this work was finishing, the paper [55] with the latest constraints on the parameter $\log_{10} B_0 < -4.1$ (2σ) has appeared. Actually, this restriction has nothing to do with our constraint because in that paper, first, no fourth sterile neutrino is considered and, second, the BZ parametrization that is formally similar to the case $n = 0$ in our notation (actually, the corresponding term in the action is even growing logarithmically with R) is used. In the other interesting article [65] it was found that the $f(R)$ model [7] increases the sum of the active neutrino masses significantly that seems to be in conflict with our consideration. In fact, the authors used another data set constraining the matter power spectrum in contrast to galaxy clusters data set constraining σ_8 used in our paper. The recent cluster mass function measurements permit a rather high value of the sterile neutrino mass without any modification of gravity due to the decrease in the value of σ_8 . The implementation of $f(R)$ gravity does not bring a significant effect here because even higher mass of sterile neutrino is forbidden by the combined CMB spectrum and BAO data set. At last, in the work [66] galaxy cluster measurements were used with the mass function enhancement according to modified gravity as compared to the Λ CDM consideration used here but once more, for the standard number of neutrino species only. In this paper, the Hu–Sawicky model [5] is used which is very similar to our model [7] and has the same behaviour at large R with $2n$ denoted by n . Thus, our results for $n = 2$ without the sterile neutrino have to be compared with theirs for $n = 4$. Earlier, N-body calculations of the non-linear matter power spectrum, the halo mass function and the halo bias with massive neutrinos were made in [67] for the Hu–Sawicky $f(R)$ model in the case $n = 1$ that corresponds to $n = 0.5$ in our model. In all papers [55, 65–67] 3 standard neutrino species were assumed.

To understand which gravity law is more preferable by cosmological data, we compare differences of logarithmic likelihoods $\log L$ calculated for different Universes with the same

data set; for the latter we chose the *ePlanck*+*BAO*+*LENS*+*H₀*+*CL* set combination. Each difference $2 \cdot \Delta \log L$ is distributed as χ^2 with an effective number of degrees of freedom equal to the difference of the numbers of fitting parameters in the two corresponding universes. The improvement of maximum likelihood for the $f(R)$ model (λ for the extra fitting parameter) with one massive sterile and three massless active neutrinos as compared to the Λ CDM model with one free massive and two massless active neutrinos (normal hierarchy pattern) is $\Delta \log L = 0.85$ which corresponds to $\chi^2 = 1.71$ for 1 degree of freedom. Significance of such improvement is about 1.3σ . Therefore, the Universe with one additional massive sterile neutrino within $f(R)$ gravity is slightly more preferable than the Λ CDM model with 3 active neutrinos (assuming normal hierarchy). On the other hand, this would not be true if sterile species were not included into the modified gravity model. Indeed, implementation of $f(R)$ gravity in the case of only three active neutrinos (with one massive) spoils the goodness-of-fit with respect to the Λ CDM model with the same neutrinos: $\Delta \log L = -3.42$ or $\chi^2 = -6.83$ for 1 degree of freedom. The reason is that the CMB+BAO data combination constrains the mass of neutrino in case of 3 active ones more tightly in comparison with the model with 4 neutrino species (see discussion in Sec. 4.2).

Moreover, $f(R)$ gravity significantly better describes the Universe with the sterile neutrino of mass ≈ 1.5 eV introduced for explanation of various anomalies in neutrino oscillation experiments, as discussed in Introduction. We compare $f(R)$ gravity with Λ CDM model for fixed sterile neutrino mass assuming active neutrinos are massless. For sterile neutrino of 1 eV we obtain a maximum likelihood ratio given by $\Delta \log L = -1.07$ which corresponds to $\chi^2 = -2.14$. It occurs because galaxy cluster mass function data permit higher (though not too high) neutrino masses very well without any gravity modifications (see Fig. 6). For sterile neutrino of 1.5 eV, we find the improvement $\Delta \log L = 9.52$ for the $f(R)$ gravity model with one additional free parameter λ beyond GR. We see that although $f(R)$ implementation is not so obvious for the 1 eV sterile neutrino, in the case of the 1.5 eV mass (if found in ground experiments indeed) modified gravity improves the goodness-of-fit significantly. According to the Akaike Information Criteria (AIC) [68], if χ^2 improves by 2 or more with a new additional free parameter, its incorporation is justified. In the case of the 1.5 eV sterile neutrino, $f(R)$ implementation yields the improvement $\Delta \chi^2 = 19.05$ for one additional free parameter.

5 Conclusion

In this work, we have reviewed $f(R)$ formalism generally and the $f(R)$ cosmological model of the present DE [7] in particular. We have used the First Iteration Approach (FIA) to describe the FLRW background evolution in $f(R)$ gravity. Precision and range of application of this method were studied in the case of the [7] model.

We used CosmoMC package with the modified MGCAMB module to investigate the role of $\mathcal{O}(1)$ eV sterile neutrino in modified gravity using up-to-date cosmological data including low- z galaxy cluster mass function measurements. We do not find strong degeneracy between the sterile neutrino mass and the parameter λ of the $f(R)$ gravity model used as suggested before. Moreover, the $f(R)$ gravity effect on different parameter constraints is not significant. Surprisingly, the existence of the sterile neutrino shifts the scalar spectral index n_s to a value very close to unity irrespective of the law of gravity used: GR or $f(R)$ gravity. More importantly, modified gravity improves the maximum likelihood significantly for the fixed sterile neutrino mass ≈ 1.5 eV which is suggested by various anomalies in neutrino oscillation experiments. Along with the fact that this modified gravity does not spoil the goodness-of-fit

for lower sterile neutrino masses, $f(R)$ gravity remains more preferable in the description of the Universe.

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